The Density Field in Extended Lagrangian Perturbation Theory

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Abstract

We showed the performance of Lagrangian perturbation theory for cosmological dynamics. We solved hydrodynamic equations for a self-gravitating fluid with pressure, given by a polytropic equation of state. Using these results, we describe density fields with scale-free spectrum, SCDM, and LCDM models. We analyze cross-correlation coefficient of the density field between N-body simulation and Lagrangian linear perturbation theory, and the probability distribution of density fluctuation. From our analyses, the case of the polytrope exponent 5/3 shows better performance than past approximations in a quasi-nonlinear regime.

1. Introduction

The Lagrangian approximation for cosmological fluids provides a relatively accurate model even in quasi-linear regime. Zel'dovich [1] proposed linear Lagrangian approximation for dust fluid. This model is called Zel'dovich approximation (hereafter ZA). ZA describes the evolution of density fluctuation better than Eulerian approximation. However ZA cannot describe the model after the formation of caustics. After that, although some modification theory was proposed, the physical origin of the modification is not clarified.

In past approximations, such as ZA and its modified version, velocity dispersion was ignored. Buchert and Domínguez [2] argued that the effect of velocity dispersion become important beyond the caustics. They also argued that models for large-scale structure should rather be constructed for a flow which describes the average motion of a multi-stream system. Then they showed that when the velocity dispersion is still small and can be considered isotropic, that gives effective 'pressure' or viscosity terms. Furthermore, they argued the relation between mass density ρ and pressure P.

Adler and Buchert [3] have formulated the Lagrangian perturbation theory for a barotropic fluid. Morita and Tatekawa [4] and Tatekawa et al. [5] solved the Lagrangian perturbation equations for a polytropic fluid ($P \propto \rho^\gamma$) up to second

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order for cases where the equations are solved easily.

We analyzed the density field which is described by the Lagrangian approximations [6]. We calculate the cross-correlation function of density field between the Lagrangian approximation and N-body simulation. Furthermore we analyze the probability distribution function of density fluctuation for confirmation. From these results, we determine a polytrope exponent in the equation of state.

2. Summary

We compared two statistical quantities between an N-body simulation and Lagrangian approximations. In our earlier work [4, 5], we solved the first-order perturbation equations in the homogeneous and isotropic background, and the second- order ones explicitly for the case $\gamma = 4/3, 5/3$ in E-dS Universe.

First, we compared these models by the cross-correlation coefficient of the density field between the N-body simulation and Lagrangian approximations. For scale-free spectrum cases, in the case of $\gamma = 5/3$, although the result slightly depends on the initial Jeans wavenumber, the pressure model shows a better performance than ZA in quasi-nonlinear regime. Furthermore, the pressure model also shows better performance than TZA.

Second, we analyzed the PDF of density fluctuation. The case $\gamma = 5/3$ shows good tendency in the PDF of density fluctuation. Although the difference of the PDF of density fluctuation between ZA and the pressure model is still small in quasi-nonlinear regime, the effect of the pressure can promote the evolution of nonlinear structure.

From analyses of the cross-correlation coefficient of density field and the PDF of density fluctuation, we can decide that it is reasonable to choose $\gamma = 5/3$ as the polytrope exponent of the equation of state.

References

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